MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2015/2016

EMT1026 – ENGINEERING MATHEMATICS II

(All Sections / Groups)

30 MAY 2016
2.30 p.m – 5.30 p.m
(3 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of 9 pages (including cover page) with 4 Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Several tables are provided in the Appendix for your reference – special discrete and continuous probability distributions, the cumulative standard normal distribution, common Laplace transform pairs and properties.

Question 1

(**Note**: The tables in APPENDIX A, APPENDIX B and APPENDIX C may be useful for solving this question.)

(a) The probability mass function, $f(x)$, of a discrete random variable, X, is tabulated below:

x	0	1	2	3
$f(x)$	h	i	j	k

(i) Find the values of h, i, j and k if it is known that $P(X \leq 2) = 0.9$, $P(X < 2) = 0.5$ and $E[X] = 1.5$.

[8 marks]

(ii) Hence, compute the variance of X.

[2 marks]

(b) An aircraft can accommodate a maximum of 72 passengers. The airline has found, from historical statistics, that approximately 4% of people who purchased tickets for a particular flight ***fail*** to show up for that flight. Suppose the airline sold 75 tickets for a particular flight, determine the probability that

(i) more than 72 people show up for that flight.

[4 marks]

(ii) there are one or more empty seats on that flight.

[3 marks]

(c) The Intelligence Quotient (IQ's) of 600 applicants of a certain college are approximately **normally** distributed with a mean of 115 and a standard deviation of 16.

(i) Estimate the number of applicants who have IQ's between 95 and 135.

[4 marks]

(ii) If the college requires an IQ score of at least 100, how many of the 600 applications will be rejected?

[4 marks]

Continued...

Question 2

(a) Find the general solution for the second-order ordinary differential equation

$$y'' - 6y' + 5y = x^2 - \frac{112}{25}.$$

Then, determine the particular solution that satisfies the initial conditions $y(0) = 0$ and $y'(0) = 0$.

[13 marks]

(b) Obtain the first four nonzero terms in the power series solution about $x_0 = 0$ of the following second-order ordinary differential equation:

$$(x+2)y'' - xy' + 4y = 0.$$

[12 marks]

Question 3

(a) Find the general solution of the heat equation

$$25u_{xx} - u_t = 0$$

in the region $0 \le x \le L$, for $t > 0$, that satisfies the homogeneous boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$. Use the method of separation of variables.

[18 marks]

(b) Consider the heat equation

$$u_t = 9u_{xx}, \quad 0 \le x \le L, \quad t > 0$$

with the boundary conditions

$$u(0,t) = K_1, \; u_x(L,t) = K_2$$

and the initial condition

$$u(x,0) = K_3,$$

where K_1, K_2 and K_3 are positive constants. By letting $u(x,t) = w(x,t) + v(x)$, decompose the given problem into a homogeneous PDE and an ODE. (**Note**: Do not solve the PDE and ODE found.)

[7 marks]

Continued...

Question 4

(**Note**: The tables in APPENDIX D and APPENDIX E may be useful for solving this question.)

(a) Using appropriate transform properties, find the Laplace transforms of the following functions:

(i) $f(t) = 1 * \cos 2t + t \sin 2t$, where $*$ denotes convolution.

(ii) $f(t) = (t-1)^2 e^{-t}$. [6 marks]

(b) Determine the inverse Laplace transform of the following s-domain function: [4 marks]

$$F(s) = \frac{1}{s^2}\left(\frac{1}{s} + \frac{s}{s^2+1}\right).$$

(c) Consider the following ordinary differential equation: [5 marks]

$y''(t) + y'(t) - 6y(t) = 5e^{2t}$, subject to $y(0) = 0$ and $y'(0) = 16$.

By applying Laplace transform, show that

$$L\{y(t)\} = Y(s) = \frac{16s - 27}{(s+3)(s-2)^2}.$$

Hence, or otherwise, obtain the complete solution of the above differential equation.

[10 marks]

Continued...

APPENDIX A:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x; n, p)$	
P.m.f.	$P(X=x)={}^{n}C_{x}p^{x}q^{n-x}$, for $x=0,1,\ldots,n$ and where $q=1-p$.
Mean	$E[X]=np$
Variance	$Var(X)=npq$
Hypergeometric Distribution, $X \sim h(x; N, n, k)$	
P.m.f.	$P(X=x)=\dfrac{{}^{k}C_{x}\times{}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x=0,1,\ldots\min(n,k)$.
Mean	$E[X]=\dfrac{nk}{N}$
Variance	$Var(X)=\dfrac{nk}{N}\left(\dfrac{N-n}{N-1}\right)\left(1-\dfrac{k}{N}\right)$
Poisson Distribution, $X \sim p(x; \lambda)$	
P.m.f.	$P(X=x)=\dfrac{\lambda^{x}e^{-\lambda}}{x!}$, for $x=0,1,\ldots$.
Mean	$E[X]=\lambda$
Variance	$Var(X)=\lambda$

Continued...

APPENDIX B:

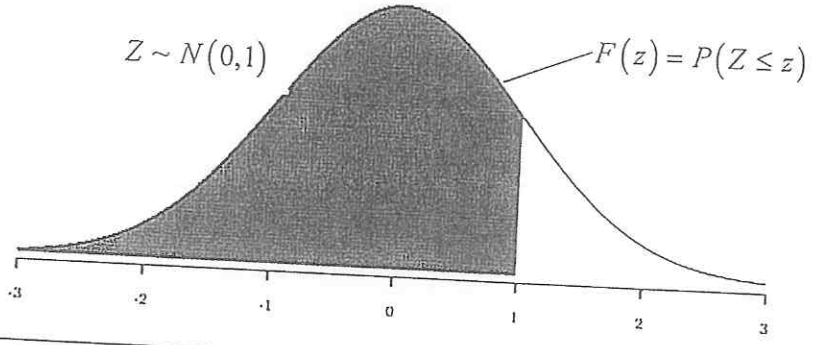
Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \dfrac{a+b}{2}$
Variance	$Var(X) = \dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$
Normal Distribution, $X \sim N(\mu,\sigma)$	
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $$Z = \frac{X-\mu}{\sigma}$$ yields a *standard* normal variable where $Z \sim N(0,1)$.	

Continued...

APPENDIX C:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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APPENDIX D:

Common Laplace Transform Pairs

	$f(t)$	$F(s)=L\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}},\ n=1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$t^{n-1}e^{at}$	$\frac{(n-1)!}{(s-a)^n},\ n=1,2,\ldots$
6.	$\cos at$	$\frac{s}{s^2+a^2}$
7.	$\sin at$	$\frac{a}{s^2+a^2}$
8.	$\cosh at$	$\frac{s}{s^2-a^2}$
9.	$\sinh at$	$\frac{a}{s^2-a^2}$
10.	$u(t-a)$	$\frac{e^{-as}}{s}$
11.	$\delta(t-a)$	e^{-as}

Continued...

APPENDIX E:

Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t)+bg(t)\} = aL\{f(t)\}+bL\{g(t)\}$
2.	s - shifting	$L\{e^{at}f(t)\} = F(s-a)$
3.	Transform of Derivative	$L\{f'(t)\} = sL\{f(t)\}-f(0)$ $L\{f''(t)\} = s^2L\{f(t)\}-sf(0)-f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}F(s)$
5.	t - shifting	$L\{u(t-a)f(t-a)\} = e^{-as}F(s)$
6.	Differentiation of Transform	$L\{t.f(t)\} = -F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\tilde{s})d\tilde{s}$
8.	Convolution Theorem	$L\{f(t)*g(t)\} = F(s).G(s)$, where $f(t)*g(t) = \int_0^t f(\tau)g(t-\tau)d\tau$.

End of paper.